FURTHER LABORATORY STUDIES OF THE ROUGHNESS AND SUSPENDED LOAD OF ALLUVIAL STREAMS

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ABSTRACT

A laboratory study was made to determine the variation with depth and velocity of the hydraulic and sediment transport characteristics of a constant-discharge flow. Eight experimental runs were performed in a 60-foot long, 33.5-inch wide recirculating laboratory flume. The unit discharge for all runs was 0.50 cfs per ft. and the velocity was varied from 0.91 to 2.21 fps, corresponding to a change in depth from 0.550 to 0.228 ft. The bed sand used for these experiments had a geometric mean sieve diameter of 0.142 mm and a geometric standard deviation of 1.38.

As the velocity was increased, the bed form changed from a dune-covered configuration to a flat bed, with sand waves occurring at intermediate velocities. It was found that for the unit discharge and bed sand used in this investigation, two different velocities and sediment transport rates are possible for a given slope, or a given bed shear velocity; however, this multiplicity is possible only in the range of slope and shear velocity where major changes in the bed configuration occur since it is a result of large variations in the bed roughness. Therefore the slope or shear velocity cannot logically be used as an independent variable since neither of these quantities uniquely determines the velocity or transport rate. However, if the velocity is used as the independent variable for a constant-discharge flow, the slope, shear velocity, and friction factor are all uniquely determined. The sediment transport rate was found to be a single-valued, uniformly increasing function of velocity, and it can therefore be used in place of the velocity as the independent variable.

A comparison of data from this investigation with data from previous investigations which used the same sand showed that even a small decrease in the amount of fine material in the bed sand can have a significant effect on the transport rate. However, even relatively large changes in the standard deviation of the bed material have a small effect on the friction factor.

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CHAPTER 1

INTRODUCTION, APPARATUS AND PROCEDURE, AND SAND CHARACTERISTICS

1-1. Background and Objectives of this Research

As suggested by the title, this report presents the results of a group of experiments performed to extend some previous research (1, 2) which was carried out at California Institute of Technology. The significance of these experiments can be best understood in the light of the previous investigations. In 1955 Brooks (1) reported a laboratory study of the mechanics of open channel flow over a sand bed. These experiments were conducted in the 40-foot long, 10.5-inch wide recirculating flume which was located in the Sedimentation Laboratory at Caltech. The results of this investigation led him to question some widely held concepts about the relation between the sediment transport rate, depth and velocity of flow, and slope of alluvial channels. Brooks' principal conclusions will be summarized in Chapter 3 in connection with the discussion of results of the present experiments.

The discussion of Brooks' paper (1) was, for the most part, quite vigorously skeptical. It was mainly directed toward questioning the validity of his experiments, disputing his interpretation of laboratory and field data, and trying to explain his conclusions in terms of existing theories. In his closing discussion (1),Brooks presented the results of a new, larger group of experiments, some of which were designed to answer specific objections raised by the discussers. The new investigation covered a wider range of velocity at two different constant depths and used three sands with size distributions different from the two sands he originally used. These runs were conducted in the 60-foot long flume which was used for the present investigation. The results of the new experiments were in complete agreement with Brooks' original conclusions with the exception of those regarding the variation of sediment transport rate, slope, and friction factor with velocity for a constant discharge. These were neither proved nor disproved because of the manner in which the runs were scheduled; the depth was fixed for each series of runs and the discharge was varied.

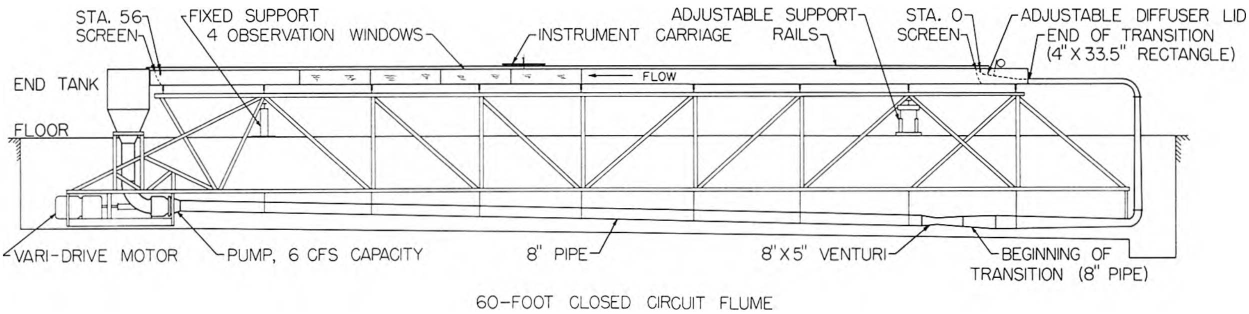
In 1957, Vanoni and Brooks (2) published a report which included the results reported in Brooks' closing discussion along with the results of some experiments on the effect of temperature on sediment transport rate, and the effect of suspended load on friction factor. The present report is intended as a supplement to the Vanoni and Brooks report.

The experiments reported in Brooks' closing discussion and later by Vanoni and Brooks included two systematic sets of runs, each with a nearly constant depth. The results of these experiments were used to evaluate Brooks' qualitative conclusions regarding the variation of sediment discharge, slope, and friction factor with velocity for constant depths, and with depth for constant velocities. As mentioned above, no systematic data were obtained for the case in which the discharge is held constant and the depth is varied; the objective of the present experiments was to verify Brooks' conclusions for this case. The main point to be resolved was whether two different depths of flow are possible for a given discharge and slope. This question was the subject of considerable controversy in the discussion of Brooks' paper and was not settled by the later experiments (1, p. 582 and p. 594). In the present inquiry, eight runs were conducted in the 60-foot flume at a constant unit discharge of 0.50 cfs per ft. The results of these experiments are presented in Chapter 2 and a discussion of the results is given in Chapter 3.

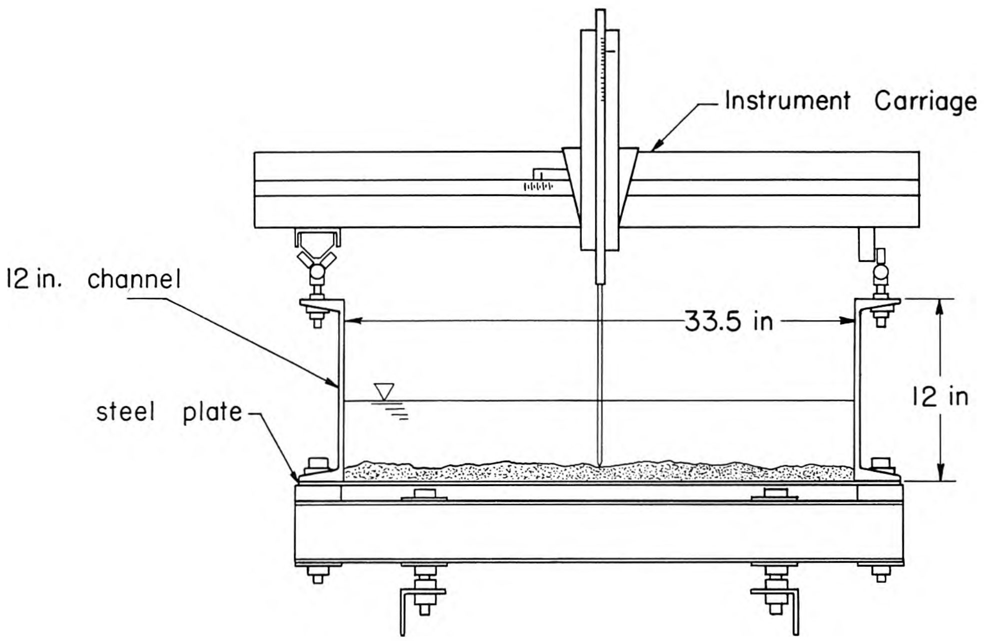
1-2. Apparatus and Procedure

Only a summary description of the apparatus and procedure will be presented here. Complete details and thorough discussions of the apparatus, the techniques used in measuring the various quantities, and the procedure used in reducing the data have been given by Brooks (1) in his original paper on this topic, and by Vanoni and Brooks (2).

The experiments were performed in the 33.5-inch wide, 60-foot long flume shown schematically in Figure 1-1. The flow circuit of the flume is closed so the water and sediment are continuously recirculated and the mean depth of flow in the open channel is controlled by the amount of water in the system. The entire flume is mounted on a truss which is pivoted near one end and supported by a jack at the other end so the flume slope can be set at any desired value and easily adjusted during



(a) Elevation



(b) Cross section of flume

Fig. 1-1. Schematic diagram of the 60-foot flume.

the course of a run. The instrument carriage is mounted on rails and can be moved along the length of the flume. A point gauge mounted on the carriage was used to measure the elevations of the water surface and sand bed relative to the carriage rails. Water surface elevations were measured during the run at four-foot intervals along the flume, and bed elevations were measured after the pump had been stopped and the bed leveled in four-foot reaches with a special scraper. The discharge was measured with the venturi meter located in the return pipe. To determine the total sediment transport rate, twelve or more samples of one liter each were siphoned through special samplers from the vertical section of pipe above the pump. The individual one-liter samples were then passed through filter paper, and the sediment retained was dried and weighed. A sieve analysis was performed on the composite of all samples for each run with a fourth-root-of-two set of half-depth Tyler laboratory sieves which were shaken for fifteen minutes on a Tyler Rotap Machine.

For Run 3-5 and subsequent runs a heater was provided to control the temperature of the water. The water temperature for these runs was maintained at approximately 25°C.

A side-wall correction procedure was applied to the measured and computed hydraulic data to determine the friction factor, shear velocity, and hydraulic radius of the sand bed section. This procedure supposedly corrects for the effects of the smooth side-walls on these quantities. Vanoni and Brooks (2) have presented a detailed derivation of the side-wall correction procedure used here and the data necessary to perform the computations.

1-3. Sand Characteristics

The sand used in this investigation was the sand designated as Sand 4 by Vanoni and Brooks (2) and used in their investigation. The sieve analysis they reported was performed on a sample of bed material which was taken about midway through their fifteen runs with this sand. Between the time their experiments were completed and the present investigation was started the sand remained in the flume, but was not used for any experiments. After the completion of the present experiments a sample of bed material was removed from the flume and sieve analyzed as described in Section 1-2. The results of the sieve analysis reported by Vanoni and Brooks and the sieve analysis performed for this investigation are summarized in Table 1-1 and shown plotted on logarithmic-probability paper in Figure 1-2.

Figure 1-2 shows that there was very little increase in the geometric mean sieve diameter, D_g, and no change in the geometric standard deviation, σ_g, of the size distribution. However, in Table 1-1 it is seen that there was a significant decrease in the amount of fine material in the bed sand. For example, at the conclusion of the present experiments there was only about 0.76 times as much material finer than the 170-mesh sieve as during the experiments by Vanoni and Brooks, and for the smaller sieve sizes the losses were even greater. Most of the loss probably occurred when the flume was periodically drained to be refilled with clean water. This coarsening of the bed material will be invoked in Chapter 3 to explain a discrepancy in the transport rate between the results of the present experiments and those of Vanoni and Brooks.

Table 1-1

Summary of Data from Sieve Analyses
by Vanoni and Brooks (2) and Kennedy

Analysis by		Vanoni and Brooks	Kennedy
Date		Sept., 1956	June, 1958
Geom Mean Diam.		D_g = 0.137 mm	D_g = 0.142 mm
Geom Std. Deviation		σ_g = 1.38	σ_g = 1.38
Mesh (Tyler) Per Inch	Sieve Opening mm	Percent Finer by Weight	
42	0.351	99.84	- -
48	0.295	99.10	- -
60	0.246	96.39	95.36
65	0.208	89.00	88.20
80	0.175	79.39	76.75
100	0.147	57.47	52.30
115	0.124	40.15	33.30
150	0.104	17.74	14.36
170	0.088	10.76	8.18
200	0.074	4.44	2.52
250	0.061	2.22	1.45
270	0.053	1.56	0.43
325	0.043	- -	0.10
400	0.038	- -	0.04

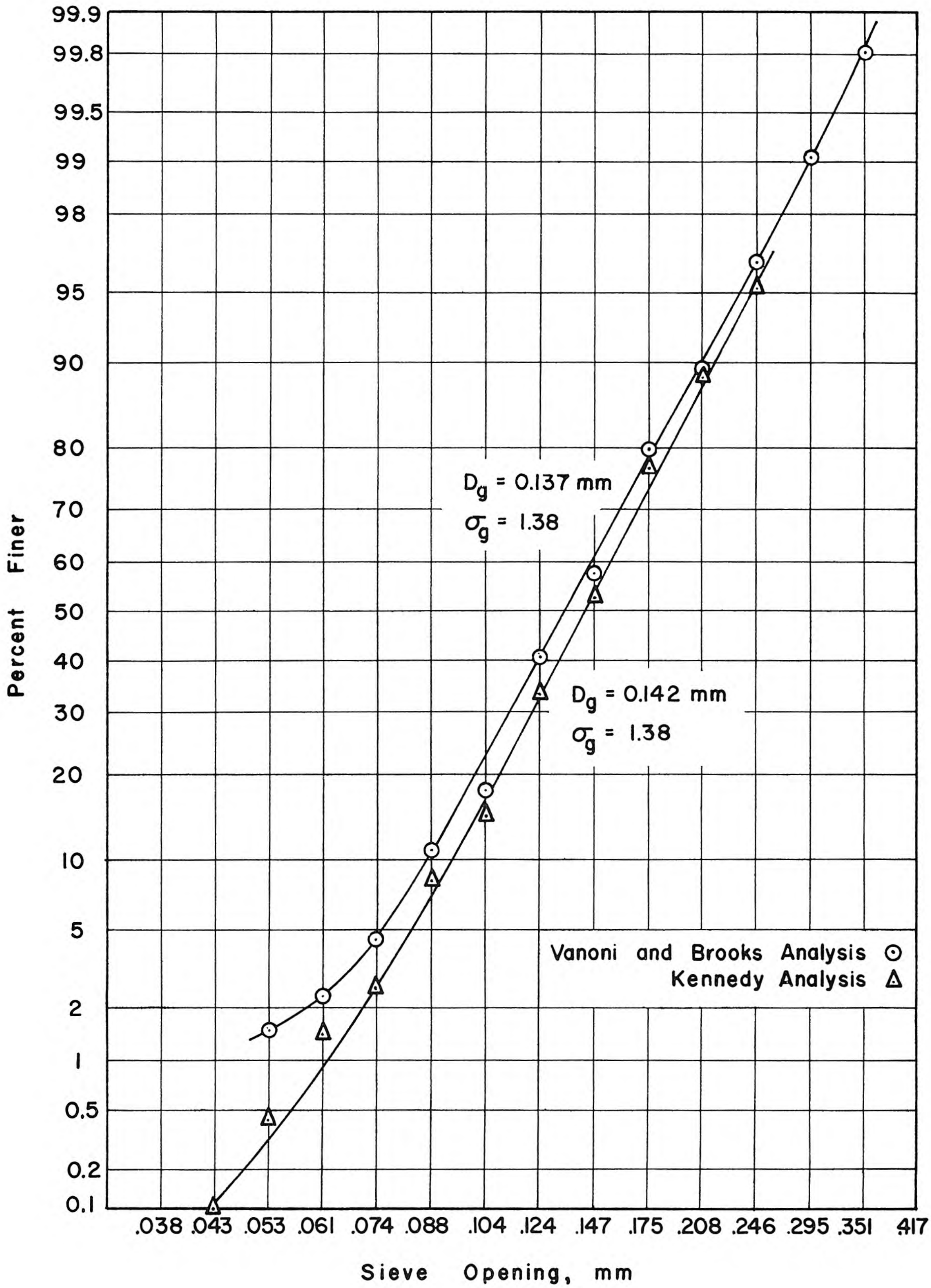


Fig. 1-2. Sieve analyses by Vanoni and Brooks (2) and Kennedy of the sand used in the present investigation.

CHAPTER 2

EXPERIMENTAL RESULTS

2-1. Summary of Experimental Results

Eight runs were conducted with a unit discharge of 0.50 cfs per ft at depths ranging from 0.228 to 0.550 ft. In each run sufficient time was allowed for the flow to reach equilibrium with its sand bed and uniform flow was established before the reported data were collected.

The principal measured and computed quantities from the laboratory experiments are summarized in Table 2-1. The runs are tabulated in order of decreasing depth and increasing velocity. The table is self-explanatory except for the quantity f_b/f'_b, the friction factor ratio. In this quantity f'_b is the friction factor predicted by the Moody pipe-friction diagram using the geometric mean sieve diameter, D_g, as the equivalent sand roughness, and $4r_b$, where r_b is the bed hydraulic radius, in place of the pipe diameter for the characteristic length. Thus f'_b is friction factor predicted by the Moody diagram for flow of clear water over a stationary flat sand bed. The friction factor ratio, f_b/f'_b is a measure of the effect of dunes or other bed features on the friction factor of the flow and serves as a partial quantitative description of the bed configuration; it should be approximately unity for flow over a flat bed. The use of the friction factor ratio in analyzing the roughness of alluvial channels was first suggested by Vanoni and Brooks (2). Detailed discussions of the friction factor ratio and its significance have been presented by Kennedy (3) and Taylor and Brooks (4).

The various bed configurations (Column 18) will be described in Section 2 of this chapter.

The data for Run 3-5, the run with a long, flat sand wave, are much less accurate than the data for the other runs. In this run there were no reaches of strictly uniform flow and the data for the dune and flat sections are averages over the reaches in which the flow was the most nearly uniform; these reaches were only about twenty feet long. In such

TABLE 2-1

SUMMARY OF HYDRAULIC AND SEDIMENT TRANSPORT DATA FROM LABORATORY EXPERIMENTS

Sand Characteristics: D_g = 0.142 mm σ_g = 1.38

Run No.	1 d Depth ft	2 U Mean Velocity ft/sec	3 q Unit Dis-charge cfs/ft	4 F Froude Number	5 r Hydr. Radius ft	6 S Slope	7 U_* Shear Velocity ft/sec	8 f Friction Factor	9 r_b Bed Hydr. Radius ft	10 U_{*b} Bed Shear Velocity ft/sec	11 f_b Bed Friction Factor	12 $\frac{f_b}{f'_b}$ Friction Factor Ratio	13 T Water Temp. °C	14 $\bar{C}$ Disch. Conc. gr/l	15 q_s Sed. Disch. Rate lb/min-ft	16 Analysis of Sediment Load D_g mm	17 σ_g	18 Bed Config.	Run No.
3-1	0.550	0.91	0.500	0.22	0.395	0.00056	0.084	0.068	0.503	0.095	0.086	4.85	19.5	0.014	0.026	0.103	1.60	Dunes	3-1
3-4	0.441	1.14	0.503	0.30	0.335	0.00145	0.125	0.096	0.414	0.139	0.119	6.60	18.4	0.39	0.73	0.100	1.46	Dunes	3-4
3-2	0.373	1.35	0.504	0.39	0.294	0.00206	0.140	0.086	0.352	0.153	0.102	5.67	18.4	1.42	2.68	0.096	1.43	Dunes	3-2
3-7	0.345	1.45	0.500	0.44	0.277	0.00198	0.133	0.067	0.324	0.144	0.079	4.34	25.1	1.13	2.12	0.097	1.47	Dunes	3-7
3-5D*	0.340	1.47	0.500	0.44	0.273	0.0016	0.119	0.052	0.315	0.126	0.060	3.30	25.3	0.98	1.84	0.085	1.45	Dunes(S.W.)	3-5D*
3-5F*	0.254	1.97	0.500	0.69	0.215	0.0025	0.131	0.036	0.234	0.137	0.039	2.05	25.3	1.71	3.20	0.102	1.49	Flat(S.W.)	3-5F*
3-6a	0.235	2.13	0.501	0.78	0.201	0.00198	0.113	0.023	0.209	0.115	0.023	1.22	25.7	1.57	2.95	0.127	1.41	Flat	3-6a
3-6	0.233	2.15	0.501	0.78	0.200	0.00207	0.115	0.023	0.209	0.118	0.024	1.25	25.2	1.41	2.65	0.128	1.38	Flat	3-6
3-6b	0.228	2.21	0.504	0.82	0.196	0.00221	0.118	0.023	0.205	0.121	0.024	1.24	25.4	1.74	3.29	- -	- -	Flat	3-6b

* Dune section of flow with a long sand wave

* Flat section of flow with a long sand wave

S.W. Sand wave developed on bed.

short reaches the depth, sediment concentration, and particularly the slope were difficult to determine.

No experiments were carried out at velocities greater than about 2.2 fps because of the large stationary surface waves which occurred at higher Froude numbers. It would have been desirable to obtain one or two more runs at higher velocities in the flat bed regime but this was impossible at the selected discharge ($q = 0.50$ cfs per ft). A larger value of q would have permitted runs to be carried out at velocities which are in the flat bed regime, but for which the Froude numbers are small enough to preclude the occurrence of surface waves. However, a larger q would have also resulted in larger depths, and thus larger depth-width ratios which are undesirable because of the effects of the side-walls. At the lower velocities, a larger q would have given depth-width ratios greater than 0.2 which was judged too large. Although the effects of the side-walls on the friction factor can supposedly be corrected for, there is no known method to determine their effect on the sediment transport characteristics of the flow.

For convenient reference, the data from other investigations that used this sand are summarized in the Appendix. The results of Vanoni and Brooks' experiments in the 60-foot flume (2, p. 36) are summarized in Table A-1, and the data from Nomicos' variable-temperature experiments in the 40-foot flume (2, p. 37) are given in Table A-2.

Figure 2-1 shows graphically the ranges of depth and unit discharge which have been covered by the experiments of Vanoni and Brooks (2) and the experiments of this investigation. All of the data shown are for the same sand and the 60-foot flume. The slope times 10^4 and the bed friction factor times 10^2 are noted by each point. The runs of Vanoni and Brooks fall about two lines of constant depth, while the runs of this investigation plot along a vertical line of constant discharge. This illustrates the difference in the way the runs of the two investigations were scheduled.

2-2. Sand Bed Configurations: Dunes, Sand Waves, and Flat Bed

Only a brief discussion of the various sand bed configurations and their occurrence will be presented. Thoroughgoing descriptions and discussions of the three bed configurations considered here have been

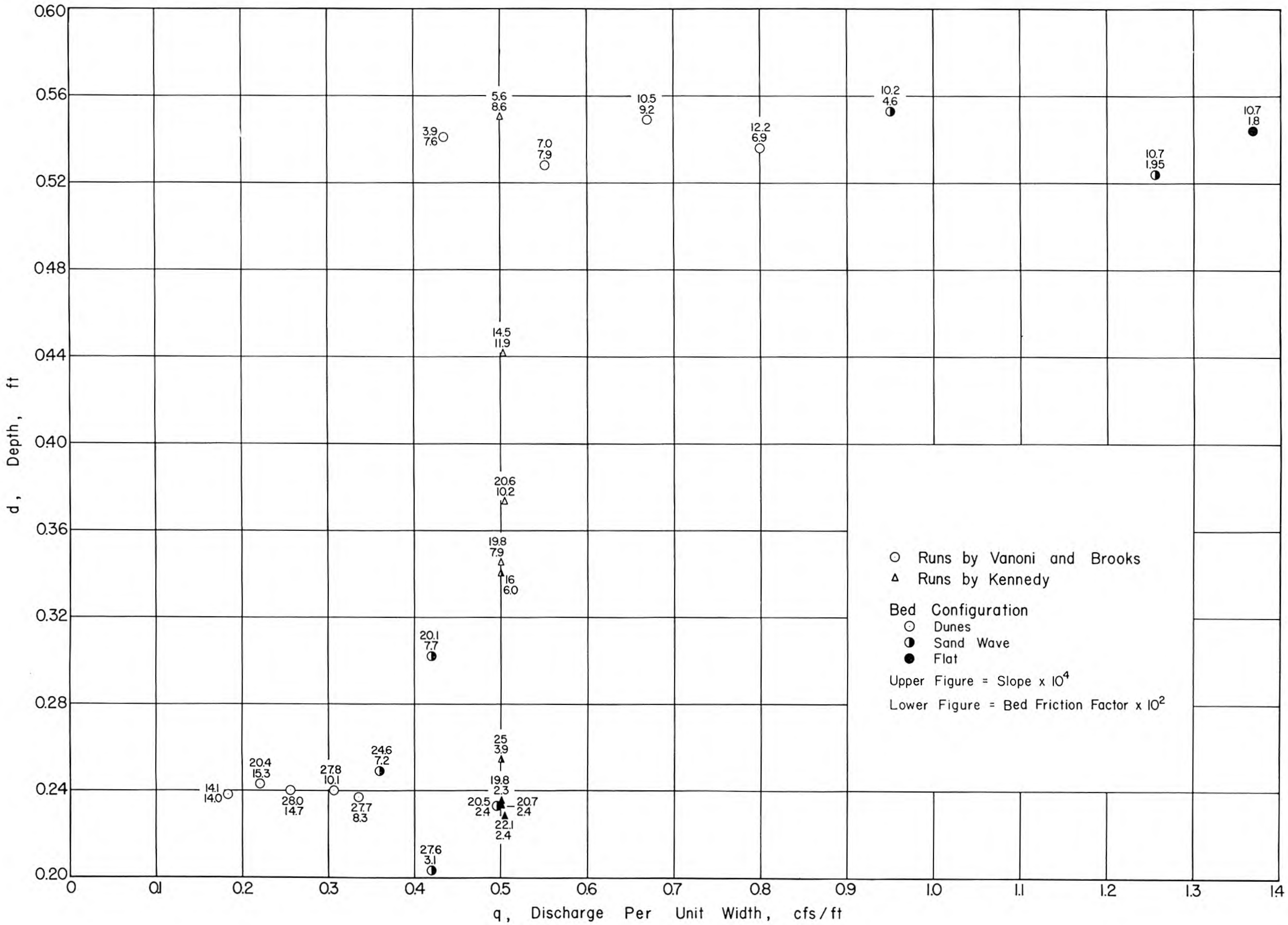


Fig. 2-1. Graphical summary of experiments by Vanoni and Brooks (2) and Kennedy in the 60-foot long flume. Sand characteristics: $D_g = 0.14$ mm; $\sigma_g = 1.38$.

given by Vanoni and Brooks (2) and Brooks (1). Treatments of the bed forms of alluvial channels have also been presented by Garde and Albertson (5) and Simons and Richardson (6, 7). The terminology used here is the same as used by Vanoni and Brooks (2).

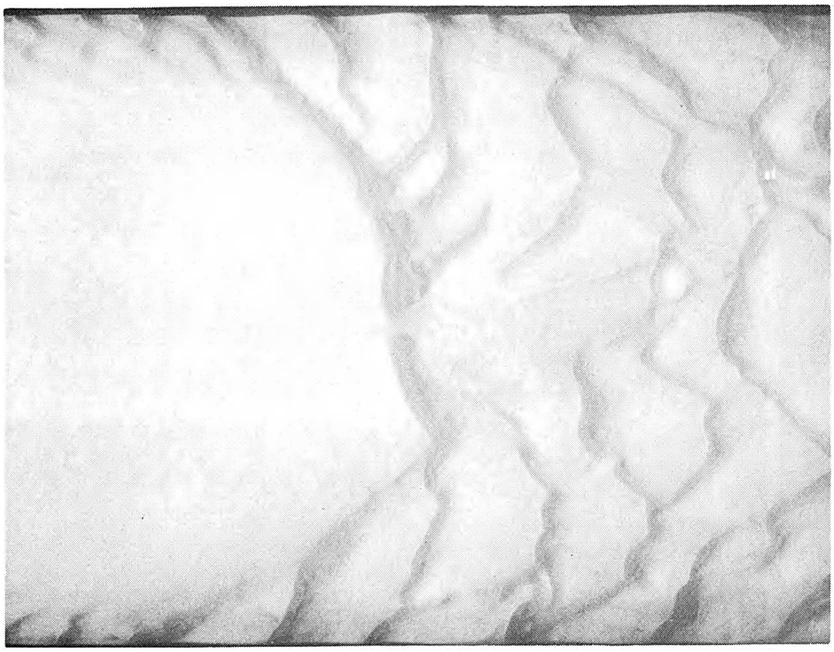
If the flow velocity is great enough to move individual sand grains, but less than another limiting value which will be discussed presently, irregular features form on the bed. These features are large, haphazardly located, short-crested, and move slowly downstream. Bed features of this type are called dunes. The dune pattern of Run 3-2 shown in Figure 2-2 illustrates fully developed dunes of the type which occurred in these experiments.

As the velocity is increased, a value will be reached above which dunes no longer occur over the full length of the flume. Over one reach of the flume the bed is flat while the rest of the bed is dune-covered. In the reach of flat bed the depth of flow is smaller, and the velocity, sediment transport rate, and sand bed elevation are greater than in the dune section. The reach of flat bed is called a sand wave. In these experiments sand waves occurred at velocities between about 1.45 and 2.1 fps. A typical sand wave front where the bed configuration changes from flat to dunes in the downstream direction is shown in Figure 2-3. The transition from dunes to flat bed at the upstream end of the sand wave is less well-defined than the transition from flat bed to dunes and occurs over a distance of several feet. In this transition the dunes become progressively less rugged in the downstream direction as the sand bed thickens and the flow velocity increases. The flow is not strictly uniform over either the flat or the dune section. The entire sand wave moves downstream at a small, nearly constant velocity. When the sand wave reaches the downstream end of the flume, it recirculates through the pump well and return pipe, reappears at the upstream end of the flume, and repeats its journey around the closed circuit. The length of the sand wave for a given sand size apparently depends on the overall average depth and velocity in the flume, and possibly the temperature. For a given amount of water in the flume, (i.e., a given mean flow depth over the flume length) the length of the sand wave increases with increasing velocity. In Run 3-5 the sand wave length varied from

Flow direction ⟶

$U = 1.35$ fps, $d = 0.373$ ft, $f_b = 0.102$, $\overline{C} = 1.42$ gr/l

Fig. 2-2. Dune configuration of Run 3-2. The full 33.5-inch width of the flume is shown.



Flow direction ⟶

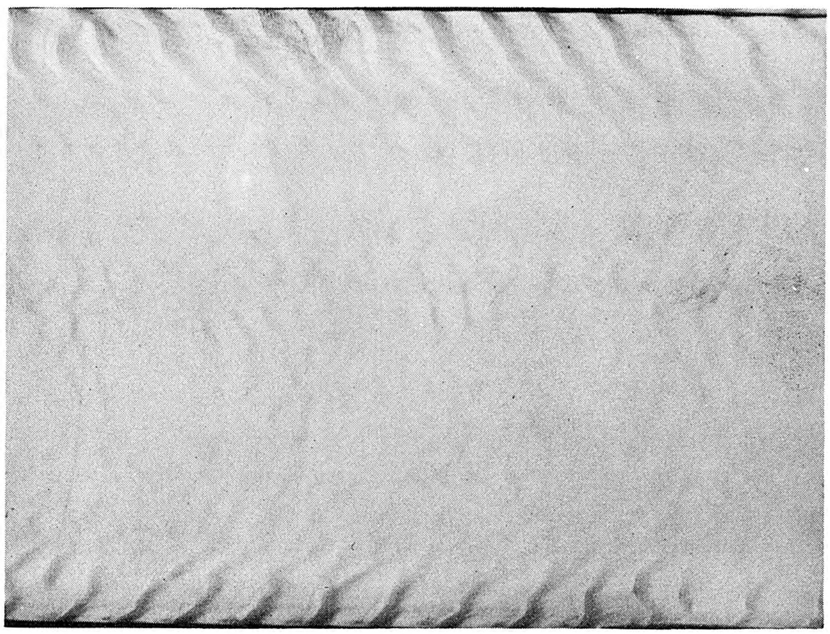
Fig. 2-3. Typical sand wave front (Vanoni and Brooks' Run 2-16). The full 33.5-inch width of flume is shown.

about ten to fifteen feet at different times in a random manner. In some preliminary runs (not reported here) two or more well-defined and apparently independent sand waves occurred simultaneously in the flume.

The mechanism involved in formation and sustenance sand waves is not well understood and they may be a result of the closed circuit arrangement of the flume. No conclusions are possible on this point with the knowledge at hand.

The third principal bed configuration observed was the flat bed. This configuration occurred at velocities greater than about 2.1 fps for the sand and unit discharge used in these experiments. The flat bed configuration of Run 3-6 is shown in Figure 2-4. The ripples near the walls are the result of the locally reduced velocity in this vicinity. For this configuration the bed is indeed very nearly flat. The variation in bed elevation relative to the carriage rails over the entire flume was usually less than a few thousandths of a foot.

At the unit discharge used in these experiments, $q = 0.50$ cfs per ft, stationary gravity waves formed at velocities greater than about 2.25 fps, which corresponds to a Froude number of 0.84. At this



Flow direction ⟶

$U = 2.15$ fps, $d = 0.233$ ft, $f_b = 0.024$, $\overline{C} = 1.41$ gr/l

Fig. 2-4. Flat bed configuration of Run 3-6. The full 33.5-inch width of the flume is shown.

velocity the coupling between the bed and water surface was sufficiently strong to produce small antidunes. At velocities greater than about 2.4 fps large antidunes and breaking stationary waves occurred. The characteristics of antidunes and stationary waves and their effects on the friction factor and sediment transport rate have been described by Kennedy (3).

CHAPTER 3

DISCUSSION OF RESULTS

3-1. Statement of Brooks' Conclusions

Much of the discussion of the present experiments will be in light of Brooks' previous findings about the mechanics of alluvial streams. For the convenience of the reader, Brooks' principal conclusions will be summarized here as follows:

1. In the laboratory flumes it was found that neither the velocity nor the sediment transport rate could be expressed as a single-valued function of the bed shear stress, or any combination of depth and slope, or bed hydraulic radius and slope. This contradicts assumptions which have commonly been held for some years to the effect that knowledge of the slope, channel geometry, and bed material of a stream were sufficient to predetermine its flow and sediment transporting characteristics.
2. The cause of the nonuniqueness cited in Conclusion (1), above, is the extreme variation in channel roughness caused by the variable nature of the bed configuration. In general, the flows at low velocities were accompanied by high, rugged dunes, while those at high velocity were associated with flat beds, with sand wave phenomena occurring at intermediate velocities.
3. On the basis of experiments to date, it appears that the depth and velocity may logically be used as independent variables along with the grain size of the bed material. Knowing these quantities, it is found for the flumes that the slope, bed shear, friction factor, and sediment discharge are all uniquely determined. It was also found possible to consider the water and sediment discharges as independent variables, and from these to determine the depth, velocity, friction factor, and slope.
4. An examination of the results of these experiments yielded the following qualitative relationships:
    a. For constant discharge, q, an increase in the sediment discharge, q_s, requires a decrease in the depth, d.
    b. For a given slope and discharge, two depths of flow are possible. When q_s is low, the bed is covered with dunes, d is large, and the velocity, U, is small. When q_s is high, the bed is flat, d is small, and U is large.

c. If q is to be increased without changing q_s, then an increase in d is necessary, although this increase is relatively less than in q.

d. For a given q, the largest values of the bed friction factor, f_b, are associated with the lowest values of q_s.

e. When U is increased with d constant, f_b generally decreases, the slope, S, and bed shear velocity, U_{*b}, may either increase or decrease, and the sediment discharge concentration, $\overline{C}$, increases until the sand wave stage is reached.

f. When d is increased with U constant, f_b and $\overline{C}$ both decrease.

g.* The bed shear velocity, U_{*b}, changes less than any other quantity. It may be expected, therefore, that U_{*b} is not a good variable from which to determine the flow and sediment transport characteristics.

h.** These conclusions are unaffected by the geometric standard deviation, σ_g, of the bed sand in the range investigated: $\sigma_g = 1.11$ to 1.76.

At a constant discharge the velocity and depth are not independent of each other but are related by $q = U d$. Thus Conclusion No. 3 implies that for a constant discharge the velocity or sediment discharge may be used as an independent variable; either of these quantities uniquely determines the slope, bed friction factor, and bed shear velocity for a given bed material.

3-2. Variation of Friction Factor and Bed Shear

The variation of slope, bed shear velocity, and bed friction factor with mean velocity are shown in Figure 3-1. In examining this graph, it should be borne in mind that the data for the run with a sand

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*Conclusion 4g is based primarily on the experiments reported by Vanoni and Brooks and was first stated in their report (2).

**Brooks original experiments used two different sands: Sand No. 1 with $D_g = 0.145$ mm and $\sigma_g = 1.11$; and Sand No. 2 with $D_g = 0.088$ mm and $\sigma_g = 1.17$. Later investigations by Vanoni and Brooks (2) extended the range of σ_g to 1.76.

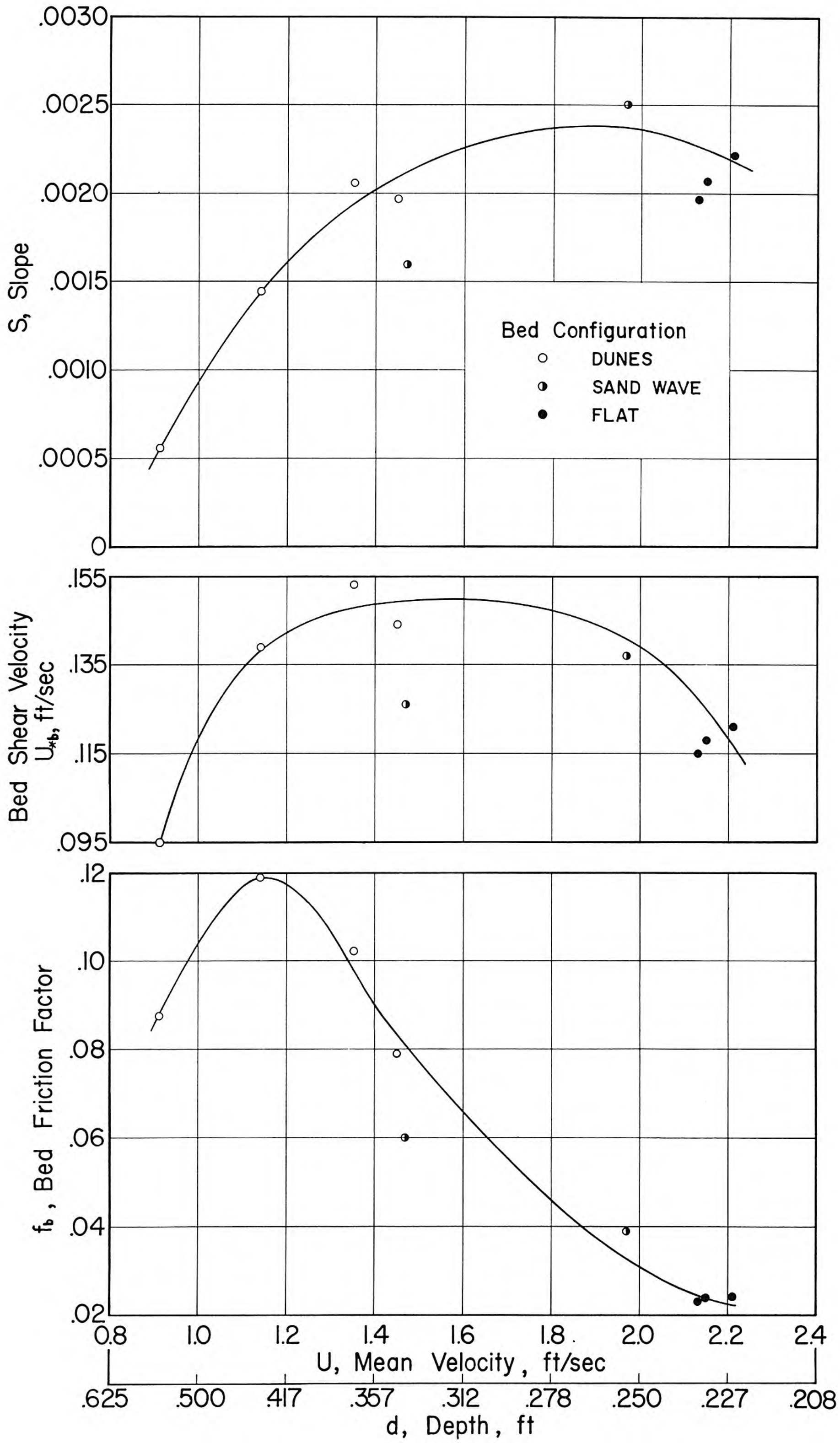

Fig. 3-1. Variation of slope, bed shear velocity, and bed friction factor with mean velocity and depth for constant-discharge experiments (q = 0.50 cfs per ft).

wave, Run 3-5, are not as accurate as the data for the other runs, as was discussed in Section 2-1. Figure A-1 in the Appendix shows a similar graph for one set of the constant-depth runs of Vanoni and Brooks summarized in Table A-1.

In Figure 3-1 it is seen that the bed friction factor increases from 0.086 to 0.119 as the velocity increases from 0.91 to 1.14 fps., corresponding to a decrease in depth from 0.550 to 0.441 ft. With further increase in velocity, the bed friction factor decreases to 0.023 at a velocity of about 2.1 fps. The initial increase of bed friction factor with velocity for this sand has been observed previously by Vanoni and Brooks (2) for depths in the range 0.524 to 0.553 ft, as shown in Figure A-1, but not for depths of approximately 0.24 ft. Thus it appears that for some depths there exists a dune arrangement which has a greater roughness than the dune configurations associated with either greater or smaller velocities. The magnitude of the maximum friction factor depends on the flow depth. The maximum f_b observed by Vanoni and Brooks in their experiments at the larger flow depths summarized in the lower part of Table A-1 was 0.092 and occurred at d =0.549 ft and U = 1.22 fps. In the present experiments the maximum bed friction factor was somewhat greater, f_b = 0.119, and occurred at a smaller depth, d = 0.441 ft, but approximately the same velocity, U = 1.14 fps.

The important point to note is that in these experiments, the bed friction factor varied by a factor greater than five from 0.119 to 0.023; at higher velocities f_b might have been even lower. The variation in f_b was due primarily to changes in the roughness of the bed as its configuration changed. As the velocity was increased above a certain value (about 1.2 fps for these experiments) the shape and arrangement of the dunes changed in such a way that the roughness of the bed decreased. When the bed became flat, the only roughness was that of the individual sand grains and was the irreducible minimum. However, a small further decrease in the friction factor could have occurred at higher velocities with larger sediment concentrations. The suspended sediment apparently modifies the turbulence structure of the flow and thereby reduces the friction factor. This effect has been described by Vanoni and Brooks (2) and Vanoni and Nomicos (8).

The large variation in the bed friction factor has an important effect on the relation between U and S, and between U and U_{*b}. As shown in Figure 3-1, for a constant discharge the slope increases with velocity throughout the dune regime. In the velocity range for which the sand wave is the characteristic bed form, the slope increases slightly and then decreases. Consequently, an identical value of slope, S = 0.00198, was observed for two different runs: Run 3-7 (U = 1.45 fps, q_s = 2.12 lb/min-ft, dune bed) and Run 3-6a (U = 2.13 fps, q_s = 2.95 lb/min-ft, flat bed). These two runs confirm, for this sand and this unit discharge, Brooks' Conclusion No. 4b that for a given discharge and slope, two different velocities are possible. Similarly two runs with different velocities and different sediment transport rates had nearly the same bed shear velocity: Run 3-4 with a dune bed which had U = 1.14 fps, q_s = 0.73 lb/min-ft, and U_{*b} = 0.139 fps; and the flat section of Run 3-5 for which U = 1.97 fps, q_s = 3.20 lb/min-ft, and U_{*b} = 0.137 fps.

It should be noted that Brooks' Conclusion No. 4b regarding the multiplicity of velocity as a function of slope for constant discharge is valid only for a certain range of velocity and a corresponding range of slope. Since this multiplicity is a result of major changes in the bed configuration, two or more values of velocity for a given slope can occur only in the range of U, and associated range of S, from slightly less than that at which the transition from dunes to sand wave occurs to slightly greater than the velocity at which the bed configuration changes from sand wave to flat. For the constant discharge experiments reported here, this range of slope is about S = 0.0020 to 0.0025. Similarly, for the constant-depth runs of Vanoni and Brooks shown in Figure A-1, U is a multiple-valued function of S for slopes between about 0.0010 and 0.0012. The multiplicity of U as a function of U_{*b} for a constant q noted in the present experiments and shown in Figure 3-1 is also limited to the range of U and corresponding range of U_{*b} where major changes in the bed configuration occur.

Thus, for the laboratory stream, the velocity for a constant discharge cannot, in general, be expressed as a single-valued function of slope or bed shear velocity. However, each of the quantities S, U_{*b}, and f_b can be expressed as a single-valued function of the velocity, or in the case of these constant discharge runs, as a function of the depth. This is consistent with Brooks' Conclusion No. 3.

For velocities between about 1.45 and 2.1 fps the nature of the variations of S, U_{*b}, and f_b is obscured by the sand waves which formed. There is some question of whether uniform equilibrium flows over a sand of this size can even exist at these intermediate velocities. When the quantity of water in the flume was set to yield a velocity between 1.45 and 2.1 fps a sand wave would form; the depth over the flat bed section would be less than desired, while over the dune section the flow was deeper, with the average of the two sections yielding approximately the desired depth. This was exactly the case in Run 3-5 for which the amount of water was adjusted to give a velocity of 1.67 fps and an average depth over the flume length of 0.30 ft.

As explained in Section 2-1, no runs were carried out at velocities higher than about 2.2 fps because of the large surface waves that formed. The results of Vanoni and Brooks' experiments indicate that with increasing velocity the bed friction factor would have remained practically constant and the slope and shear velocity would have increased accordingly.

The bed shear velocity, U_{*b}, varied from a minimum of 0.095 fps to a maximum of 0.153 fps, a factor of only 1.61. The maximum f_b was over five times greater than the minimum, and the slope varied by a factor of approximately four in the velocity range investigated. This comparison supports Brooks' Conclusion No. 4g that U_{*b} is the least sensitive parameter that can be used to determine the hydraulic and transport characteristics of a flow.

The change in bed configuration from dunes and flat bed occurred at a value of f_b/f'_b between 2.05 and 3.30; the exact value is obscured by the sand wave. Taylor and Brooks (4) have analyzed the flume data of various investigators and found that the transition from dunes to flat bed usually occurs at approximately $f_b/f'_b = 2.0$.

3-3. Sediment Transport Rate

In Figure 3-2 values of sediment discharge concentration, $\overline{C} = q_s/q$, are plotted against mean velocity. Corresponding values of the sediment discharge per unit width, q_s, are shown on the ordinate scale at the right of the figure. The water temperature during the run is noted by each point.

After Run 3-4 was completed, a heater was installed on the flume to control the water temperature. For Run 3-5 and successive runs the water temperature was maintained at approximately 25°C, which was also the temperature for most of the runs reported by Vanoni and Brooks (2). Runs 3-1, 3-2 and 3-4 were performed before the heater was installed and had somewhat lower temperatures.

This temperature change is no doubt responsible for the apparent discontinuity in the velocity-sediment concentration relation between Run 3-2 ($U = 1.35$ fps, $\overline{C} = 1.42$ gm/l, $T = 18.4$°C) and Run 3-7 ($U = 1.45$ fps, $\overline{C} = 1.13$ gm/l, $T = 25.1$°C). The results of experiments by Nomicos reported by Vanoni and Brooks (2) and summarized in Table A-2 of the Appendix show that for a constant discharge, $\overline{C}$ and q_s generally decrease with increasing temperature. The drop in $\overline{C}$ between Run 3-2 and Run 3-7 is of the magnitude that Nomicos' experiments would indicate. Therefore, to put all transport data on a quantitatively comparable basis, the concentrations and sediment discharges for the three lowest velocity runs should be reduced about 25 to 35 percent.

With this adjustment (not shown), Figure 3-2 indicates that $\overline{C}$ and q_s are single-valued functions of velocity for this constant discharge and that as the velocity is increased, these quantities also increase. This is consistent with Brooks' Conclusion No. 4a that for a constant discharge an increase in q_s requires a decrease in d. However, this uniform increase of q_s with U, and the initial increase of f_b with U

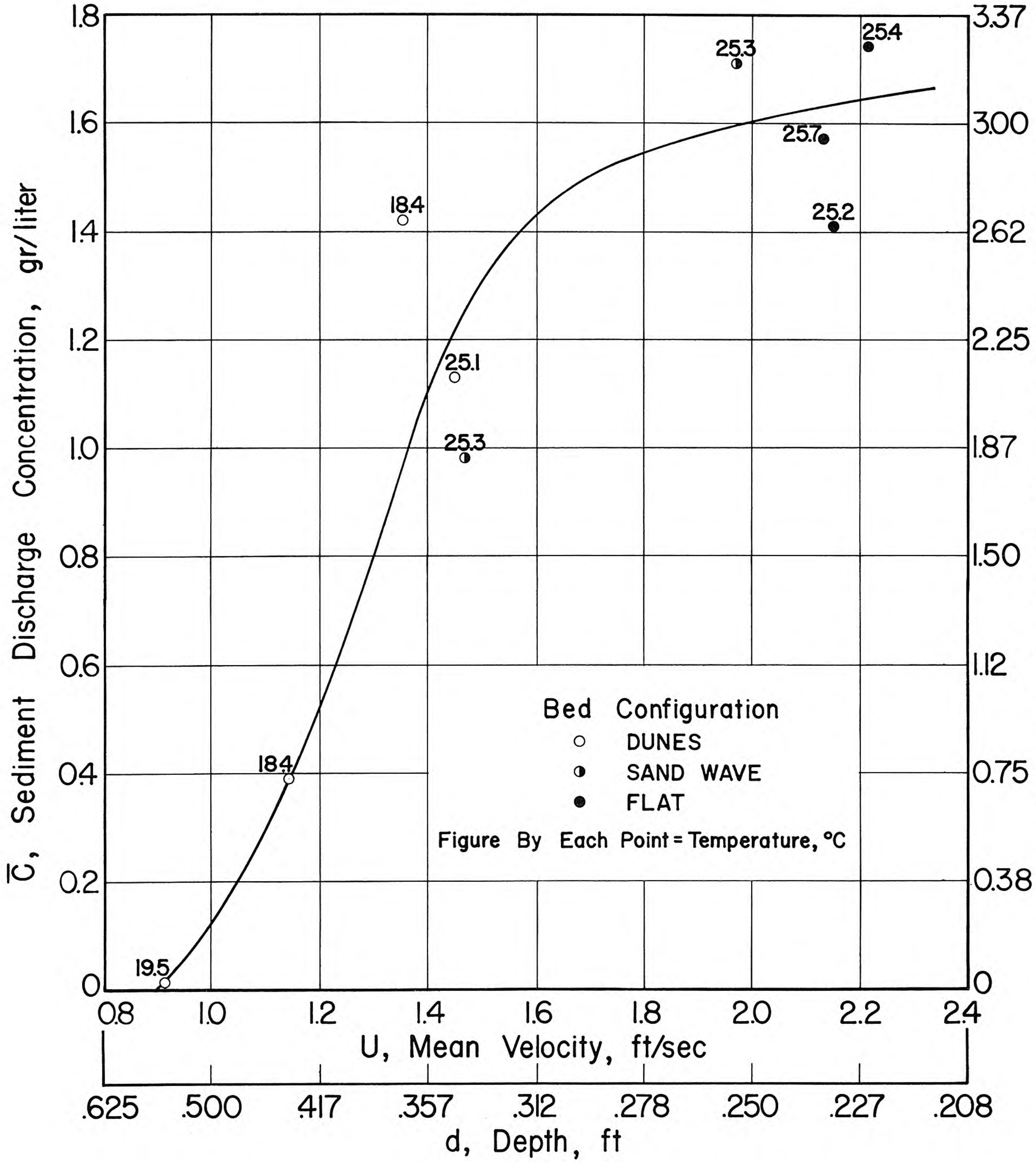

Fig. 3-2. Variation of sediment discharge concentration and sediment transport rate with mean velocity and depth for constant discharge experiments (q = 0. 50 cfs per ft).

discussed in Section 3-2, contradict Brooks' Conclusion 4d that for a given q, the largest values of f_b are associated with the lowest values of q_s.

The constant-depth experiments of Vanoni and Brooks (2) and those of Kennedy (3) have shown that for velocities in the flat bed and antidune regimes, $\overline{C}$ and q_s increase monotonically with U. This indicates that if runs with higher velocities could have been made in the present set of experiments, the sediment discharge concentration would have continued to increase with velocity. Thus the above conclusion that U uniquely determines $\overline{C}$ for a constant q is probably generally true for higher velocities also.

3-4. Comparison with Results from other Investigations

The values of $\overline{C}$ and q_s for Runs 3-6, 3-6a, and 3-6b do not agree at all well even though these runs had nearly identical depths, velocities, and temperatures and each had a flat bed. Further, the summary given in Table 3-1 of other experiments which had nearly the same depth and velocity as Runs 3-6 shows a general lack of agreement in the transport data (Columns 13 and 14). Some of the apparent discrepancies can be logically explained. For example, in Nomicos' Runs D_1, A, D, and D_2, all of which used the same bed material and were performed consecutively, the sediment discharge rate generally increases with decreasing temperature, as would be expected. Similarly, the large sediment discharge rate of Nomicos' Run 1, compared to other runs with nearly the same T and D_g, is due to a greater availability of finer, more easily transported material in the bed sand because of the much larger standard deviation of its size distribution; this also has the effect of lowering the D_g of the sediment load. The large q_s of Brooks' Runs 21 and 21a is attributable to the small size of the bed material.

Other apparent discrepancies can be explained by a comparison of the transport rates of the different sand sizes. For example, the large discrepancy between the values of $\overline{C}$ and q_s for Runs 3-6, 3-6a, and 3-6b of this investigation and Vanoni and Brooks Run 2-2 was primarily due to the slight coarsening of the bed material discussed in Section 1-3.

TABLE 3-1

SUMMARY OF RESULTS OF EXPERIMENTS BY AUTHOR AND OTHER INVESTIGATORS

Depth = 0.228 to 0.252 ft Velocity = 2.04 to 2.21 fps

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
			d	U	S	U_*	f	r_b	U_{*b}	f_b	T	$\overline{C}$	q_s						
Experiment by	Ref.	Run No.	Depth ft	Mean Velocity ft/sec	Slope	Shear Velocity ft/sec	Friction Factor	Bed Hydr. Radius ft	Bed Shear Velocity ft/sec	Bed Friction Factor	Water Temp. °C	Disch. Conc. gr/l	Sed. Disch. Rate lb/min-ft	Analysis of Sediment Load D_g mm	σ_g	Analysis of Bed Material D_g mm	σ_g	Bed Config.	Flume*
Author	Table 2-1	3-6a	0.235	2.13	0.00198	0.113	0.023	0.209	0.115	0.023	25.7	1.57	2.95	0.127	1.41	0.142	1.38	Flat	60-foot
Author	Table 2-1	3-6	0.233	2.15	0.00207	0.115	0.023	0.209	0.118	0.024	25.2	1.41	2.65	0.128	1.38	0.142	1.38	Flat	60-foot
Author	Table 2-1	3-6b	0.228	2.21	0.00221	0.118	0.023	0.205	0.121	0.024	25.4	1.74	3.29	--	--	0.142	1.38	Flat	60-foot
Vanoni and Brooks	(2) Table 6**	2-2	0.233	2.13	0.00205	0.115	0.0235	0.208	0.117	0.024	23.5	2.5	4.7	0.095	1.60	0.137	1.38	Flat (S. W.)	60-foot
Nomicos	(2) Table 7	H-2	0.233	2.13	0.0025	0.111	0.0215	0.167	0.116	0.0235	24.3	2.3	4.3	0.155	1.35	0.137	1.38	Flat	40-foot
Nomicos	(2) Table 7	1	0.241	2.06	0.00225	0.106	0.0215	0.171	0.112	0.0235	25.0	3.4	6.4	0.085	2.3	0.152	1.76	Flat	40-foot
Nomicos	(2) Table 8	D_1	0.240	2.07	0.0024	0.110	0.022	0.170	0.115	0.025	15.3	3.24	6.0	--	--	0.145	1.30	Flat	40-foot
Nomicos	(2) Table 8	A	0.241	2.06	0.0021	0.103	0.020	0.166	0.106	0.021	25.0	1.87	3.4	--	--	0.145	1.30	Flat	40-foot
Nomicos	(2) Table 8	D	0.241	2.06	0.0023	0.107	0.022	0.171	0.113	0.024	25.0	2.14	4.0	--	--	0.145	1.30	Flat	40-foot
Nomicos	(2) Table 8	D_2	0.242	2.05	0.0022	0.105	0.021	0.173	0.111	0.023	38.0	1.66	3.1	--	--	0.145	1.30	Flat	40-foot
Kennedy	(3) Table 4-2	4-1	0.236	2.10	0.0026	0.113	0.233	0.173	0.121	0.0264	30.1	2.44	4.53	0.193	1.54	0.233	1.47	Flat	40-foot
Kennedy	(3) Table 4-2	4-19	0.252	2.13	0.0021	0.120	0.0255	0.227	0.124	0.0268	24.2	1.17	2.36	0.150	1.48	0.233	1.47	Flat	60-foot
Brooks	(2) Table 15	4	0.236	2.08	0.0024	0.108	0.022	0.164	0.112	0.023	12.5	2.45	4.5	0.141	1.13	0.145	1.11	Flat	40-foot
Brooks	(2) Table 15	7	0.243	2.04	0.0021	0.103	0.020	0.170	0.107	0.0225	31.5	2.15	4.0	0.149	1.09	0.145	1.11	Flat	40-foot
Brooks	(2) Table 15	21	0.236	2.10	0.00225	0.106	0.0205	0.166	0.110	0.022	25.0	4.85	9.0	0.078	1.28	0.088	1.17	Flat	40-foot
Brooks	(2) Table 15	21a	0.236	2.10	0.0022	0.104	0.020	0.165	0.108	0.0215	25.0	4.9	9.1	0.079	1.22	0.088	1.17	Flat	40-foot

* The 60-foot long flume is shown in Figure 1-1.
The 40-foot long flume is 10.5 inches wide.

** This run also summarized in Table A-1 of the Appendix.

In Table 3-2 the transport rate of the individual sieve fractions are tabulated for Runs 3-6a and 2-2. The cumulative transport rates of the material coarser than each sieve size are also shown. For the material coarser than the 0.124 mm sieve the differences in the transport rates of the individual sieve fractions are insignificant in view of the temperature difference between the two runs, but for the finer sieve fractions the differences are striking. For each of these finer sieve fractions, significantly more material was transported in Run 2-2 than in Run 3-6a. This was because of the greater availability of fine material in the bed sand of Run 2-2, as shown in the comparison of the sieve analyses of the bed materials in Table 1-1 and Figure 1-2. The geometric mean sieve diameter, D_g, of the transported material of Run 2-2 is accordingly much smaller and the geometric standard deivation, σ_g, is larger due to the extension of the frequency distribution toward the smaller sizes.

These two experiments point out the important effect the sand size distribution has on the sediment transport rate of the flow, and thus the necessity of accurately knowing the size distribution of the bed material in analyzing, interpreting, and comparing the transport data for sediment-laden flows.

The only explanation offered for the discrepancies in $\overline{C}$ and q_s between consecutive runs of the same investigation, such as Runs 3-6, 3-6a, and 3-6b of this investigation, and Nomicos' Runs A and D_1 involves the fact that for this depth, velocity, and sand size, the flow is just outside the highly unstable sand wave regime. It is possible that an instability similar to a sand wave existed in these runs so that although the bed appeared flat everywhere, the flow was deeper in one section than another and the sediment discharge rate was not constant along the length of the flume. The bed profiles for these runs neither proved nor disproved this speculation. The bed profiles of Kennedy's Runs 4-1 and 4-19 gave a strong indication that sand waves were present, and in Vanoni and Brooks' Run 2-2 a sand wave was actually observed. If an instability was present, the measured $\overline{C}$ would have depended on the position of the sand wave with respect to the end of the flume where the samples were taken and this would account for the differences.

Table 3-2

Transport Rate of Individual Sieve Fractions

Analysis of Sediment Load	Run 2-2 Vanoni and Brooks			Run 3-6a Kennedy		
	D_g = 0.095			D_g = 0.127		
	σ_g = 1.60			σ_g = 1.41		
Bed Friction Factor	f_b = 0.024			f_b = 0.023		
Temperature	T = 23.5°C			T = 25.7°C		
Sieve Opening mm	Percent Retained	Sediment Discharge of Sieve Fraction lb/min-ft	Cumulative Discharge of Coarser Material lb/min-ft	Percent Retained	Sediment Discharge of Sieve Fraction lb/min-ft	Cumulative Discharge of Coarser Material lb/min-ft
0.246	0.91	0.04	0.04	2.80	0.08	0.08
0.208	2.28	0.11	0.15	4.91	0.14	0.22
0.175	4.02	0.19	0.34	7.61	0.22	0.44
0.147	10.79	0.50	0.84	18.76	0.55	0.99
0.124	11.93	0.55	1.39	17.87	0.53	1.52
0.104	20.22	0.94	2.33	23.15	0.68	2.20
0.088	8.04	0.37	2.70	10.41	0.31	2.51
0.074	15.61	0.73	3.43	9.42	0.28	2.79
0.061	9.39	0.44	3.87	2.30	0.07	2.86
0.053	4.47	0.21	4.08	1.89	0.06	2.92
0.043	6.42	0.30	4.38	0.67	0.02	2.94
Pan	5.92	0.28	4.66	0.21	0.01	2.95
Sum	100.00	4.66	- -	100.00	2.95	- -

Although some of the transport data in Table 3-1 are at variance, the values of friction factor for runs which had bed material with approximately the same D_g are in very good agreement. The friction factors for Kennedy's Runs 4-1 and 4-19 are slightly larger than the others due to the larger D_g of the bed material which gave a rougher surface. Similarly, Brooks' Runs 21 and 21a had lower value of f_b due to the small size of the bed material. Thus the effect, if any, of the variances in the sediment transport rates and changes in σ_g and T on the friction factor were not significant in these experiments, and the reproducibility of f_b was excellent. Further, the width of the flume appears to have no measurable effect on f_b.

CHAPTER 4

SUMMARY AND CONCLUSIONS

The present experiments corroborate, for this sand and unit discharge, Brooks' Conclusion No. 4b (see Section 3-1) that for a given slope and discharge, two different depths of flow are possible. This conclusion is valid, however, only for slopes in the range where major changes in the bed configuration occur. It was also found that for a constant discharge, the velocity cannot in general be expressed as a single-valued function of shear velocity. However, in the range investigated, if the velocity is used as the independent variable, the slope, bed friction factor, and bed shear velocity are uniquely determined; i.e., they are single-valued functions of velocity.

For these runs at a constant discharge, an increase in the sediment discharge required an increase in the velocity, and a corresponding decrease in the depth. This is in agreement with Brooks' Conclusion No. 4a. The results of these experiments qualify Brooks' Conclusion No. 4d that the largest bed friction factors are associated with the smallest sediment transport rates, because with increasing velocity, the bed friction factor first increased slightly, then decreased. However, the sediment discharge increased steadily with increasing velocity.

The bed shear velocity, U_{*b}, changed much less than either the bed friction factor or the slope as stated in Brooks' Conclusion 4g; thus it appears that U_{*b} is not a good parameter from which to determine hydraulic and sediment transport characteristics of a flow.

A comparison of the results from the current experiments and the experiments of other investigators showed that even very slight changes in the size distribution of the bed material can have a significant effect on the sediment discharge rate and the size distribution of the transported material. The effect, if any, of the geometric standard deviation of the bed material on the friction factor is much smaller.

ACKNOWLEDGMENTS

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APPENDIX

RELATED FLUME DATA REPORTED BY VANONI AND BROOKS (Ref. 2)

A summary of the data from some previous flume experiments which are cited in the text is presented here for the convenience of the reader. Table A-1 gives the data from the two groups of constant-depth experiments performed by Vanoni and Brooks using the same sand and flume as the present study. In Figure A-1, the slope, bed shear velocity, and bed friction factor for one group of these runs are shown plotted against mean velocity. Table A-2 presents the data from Nomicos' variable-temperature experiments in the 40-foot long, 10.5-inch wide flume.

TABLE A-1

SUMMARY OF CONSTANT-DEPTH EXPERIMENTS BY VANONI AND BROOKS (Ref. 2)

Sand Characteristics: D_g = 0.137 mm σ_g = 1.38

Sand No. 4, D_g = 0.137 mm, σ_g = 1.38																			
	Q	d	r	S	U_*	U	f	T	r_b	U_{*b}	f_b		$\bar{C}$	G			F		
Run No.	Dis-charge	Depth	Hydr. Radius	Slope	Shear Vel.	Ave. Vel.	Frict. Factor	Water Temp.	Bed Hydr. Radius	Bed Shear Vel.	Bed Frict. Factor	No. of Sed. Disch. Samples	Sed. Disch. Conc.	Sed. Dis-charge	Analysis of Sediment Load D_g	Analysis of Sediment Load σ_g	Froude No.	Bed Condition	Run No.
	cfs	ft	ft		ft/sec	ft/sec		°C	ft	ft/sec			gr/l	lb/min	mm				
Depth Range: 0.203-0.302 ft																			
2-9	0.510	0.238	0.203	0.00141	0.096	0.77	0.124	23.4	0.230	0.102	0.140	20	0.037	0.071	0.086	1.44	0.28	Dunes	2-9
2-3	0.615	0.243	0.207	0.00204	0.117	0.90	0.133	24.5	0.238	0.125	0.153	13	0.24	0.54	0.094	1.53	0.32	Dunes	2-3
2-8	0.715	0.240	0.205	0.00280	0.136	1.07	0.129	25.2	0.233	0.145	0.147	16	1.15	3.1	0.096	1.49	0.38	Dunes	2-8
2-1	0.855	0.240	0.205	0.00278	0.135	1.28	0.090	25.5	0.231	0.144	0.101	12	1.9	6.0	0.086	1.56	0.46	Dunes	2-1
2-7	0.930	0.237	0.203	0.00277	0.134	1.40	0.074	22.4	0.227	0.142	0.083	20	2.2	7.6	0.086	1.54	0.51	Dunes	2-7
2-6	1.00	0.249	0.211	0.00246	0.129	1.44	0.064	27.4	0.236	0.137	0.072	8	1.4	5.3	0.086	1.64	0.51	Dunes (s.w)	2-6
2-17D*	1.17	0.302	0.248	0.00201	0.127	1.39	0.067	18.9	0.284	0.136	0.077	4	2.2	9.8	0.092	1.49	0.44	Dunes (s.w)	2-17D*
2-17F*	1.17	0.203	0.177	0.00276	0.125	2.07	0.029	18.9	0.186	0.133	0.031	8	3.0	13	0.097	1.46	0.81	Flat (s.w)	2-17F*
2-2	1.38	0.233	0.200	0.00205	0.115	2.13	0.0235	23.5	0.208	0.117	0.024	22	2.5	13	0.095	1.60	0.78	Flat (s.w)	2-2
Depth Range: 0.524-0.553 ft																			
2-12	1.21	0.541	0.390	0.00039	0.070	0.80	0.061	24.6	0.488	0.078	0.076	16	0.0033	0.015	0.097	1.66	0.19	Dunes	2-12
2-5	1.54	0.528	0.383	0.00070	0.092	1.04	0.063	23.4	0.480	0.104	0.079	16	0.068	0.39	0.085	1.60	0.25	Dunes	2-5
2-10	1.87	0.549	0.394	0.00105	0.116	1.22	0.072	21.9	0.505	0.131	0.092	16	0.21	1.5	0.084	1.49	0.29	Dunes	2-10
2-11	2.23	0.536	0.387	0.00122	0.123	1.49	0.055	25.2	0.485	0.138	0.069	16	0.67	5.6	0.078	1.52	0.36	Dunes	2-11
2-13D*	2.65	0.553	0.396	0.00102	0.119	1.72	0.038	20.7	0.482	0.126	0.046	4	1.45	14	0.088	1.50	0.41	Dunes (s.w)	2-13D*
2-16F*	3.50	0.524	0.381	0.00107	0.115	2.39	0.0185	16.5	0.408	0.119	0.0195	-	-	-	-	-	0.59	Flat (s.w)	2-16F*
2-4	3.84	0.544	0.391	0.00107	0.116	2.53	0.0170	24.9	0.416	0.120	0.0180	16	1.15	16.5	0.124	1.45	0.60	Flat	2-4

* D Dune section in runs with a long sand wave.

* F Flat section in runs with a long sand wave.

s.w. Sand wave(s) in system.

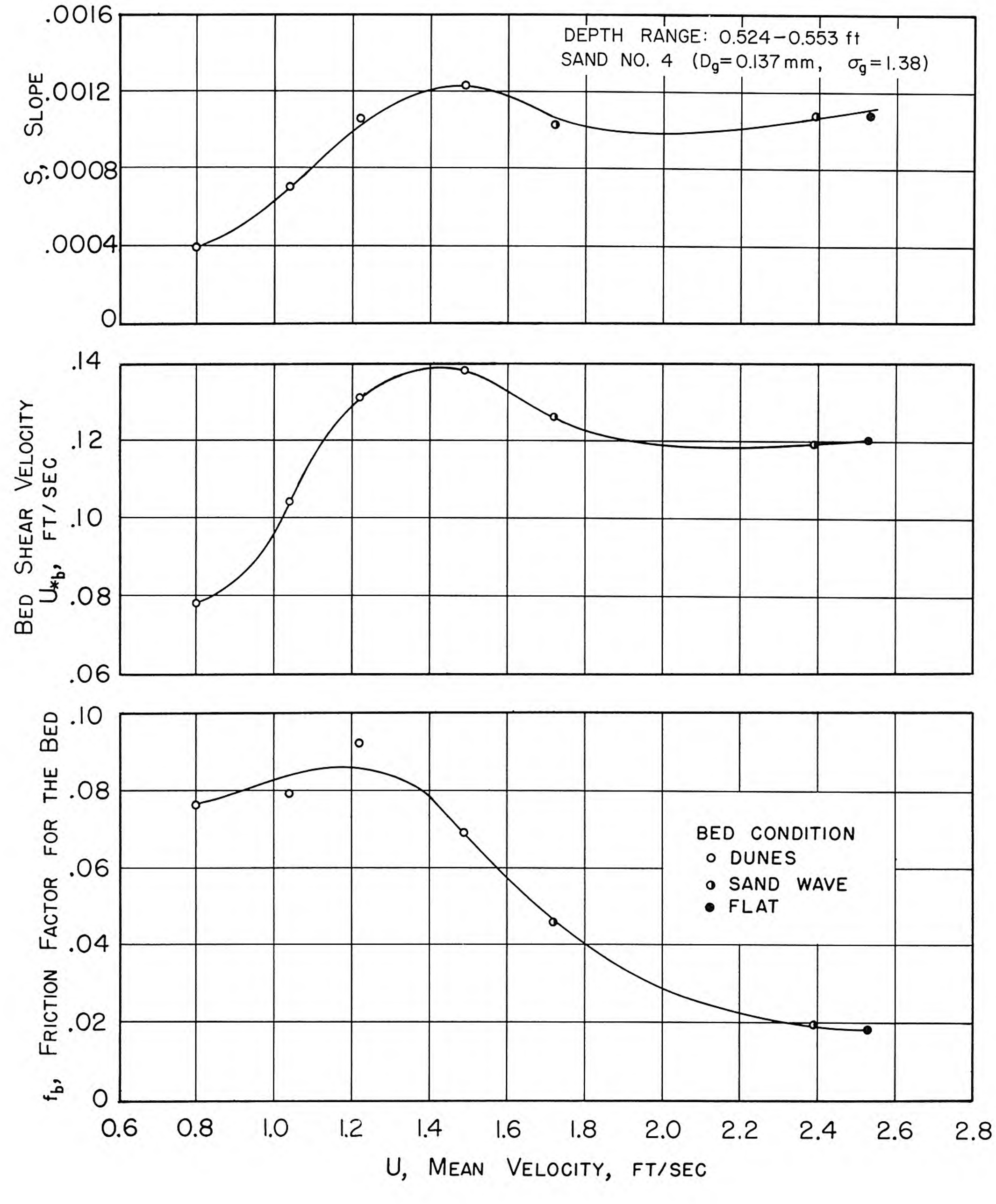

Fig. A-1. Variation of slope, bed shear velocity, and bed friction factor with mean velocity for one group of Vanoni and Brooks (2) constant-depth experiments. See Table A-1 for data.

TABLE A-2

SUMMARY OF VARIABLE-TEMPERATURE EXPERIMENTS BY NOMICOS (Ref. 2)

Sand Characteristics: D_g = 0.145 mm σ_g = 1.30

Run No.	Q Dis-charge cfs	d Depth ft	r Hydr. Radius ft	S Slope	U_* Shear Velocity ft/sec	U Ave. Velocity ft/sec	f Friction Factor	T Water Temp. ^oC	r_b Bed Hydr. Radius ft	U_{*_b} Bed Shear Velocity ft/sec	f_b Bed Friction Factor	No. of Sed. Disch. Samples	$\overline{C}$ Sed. Disch. Conc. gr/ℓ	G Sed. Dis-charge lb/min	F Froude No.	Bed Condition
D_1	0.435	0.240	0.155	0.0024	0.110	2.07	0.022	15.3	0.170	0.115	0.025	6	3.24	5.27	0.74	Flat
A	0.435	0.241	0.156	0.0021	0.103	2.06	0.020	25.0	0.166	0.106	0.021	9	1.87	3.0	0.74	Flat
D	0.435	0.241	0.156	0.0023	0.107	2.06	0.022	25.0	0.171	0.113	0.024	6	2.14	3.48	0.74	Flat
D_2	0.435	0.242	0.156	0.0022	0.105	2.05	0.021	38.0	0.173	0.111	0.023	6	1.66	2.70	0.73	Flat
G_1	0.193	0.242	0.156	0.00235	0.108	0.91	0.114	15.0	0.229	0.129	0.162	6	0.31	0.22	0.33	Dunes
C	0.193	0.241	0.156	0.0021	0.103	0.91	0.101	25.0	0.218	0.121	0.141	5	0.23	0.17	0.33	Dunes
G	0.193	0.241	0.156	0.0021	0.103	0.91	0.101	25.0	0.218	0.122	0.142	6	0.22	0.16	0.33	Dunes
G_2	0.193	0.240	0.155	0.0019	0.097	0.92	0.090	35.6	0.216	0.115	0.126	7	0.11	0.08	0.33	Dunes

SUMMARY OF NOTATION

A = cross sectional area of the stream.

$\overline{C}$ = sediment discharge concentration = q_s/q.

d = depth of flow.

D_g = geometric mean sieve diameter.

f = Darcy-Weisbach friction factor for channel = $8 (\frac{U_*}{U})^2$.

f_b = friction factor for bed section only calculated from side-wall correction procedure = $8 (\frac{U_{*b}}{U})^2$.

f'_b = friction factor determined from pipe-friction diagram using $4\ r_b$ in place of the pipe diameter as the characteristic length and D_g as the equivalent sand roughness.

F = Froude number = $\frac{U}{\sqrt{gd}}$.

g = gravitational acceleration.

p = wetted perimeter of the stream.

q = discharge per unit width = $U\,d$.

q_s = total sediment discharge rate per unit width.

Q = total discharge.

r = hydraulic radius = A/p.

r_b = hydraulic radius of the bed section calculated from side-wall correction procedure.

S = slope of energy grade line.

T = water temperature.

U = mean velocity = Q/A.

U_* = shear velocity for whole channel = $\sqrt{grS}$.

U_{*b} = shear velocity for bed section only = $\sqrt{gr_bS}$.

σ_g = geometric standard deviation of sand size distribution.

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